The Relationship Between Peak Flow and Flood Volume for Deriving Probable Maximum Flood Hydrographs in Ontario

Dequan Zhou[1] and Diana Sankar[2]
Hatch Ltd. Ontario, Canada[1]
E-mail: dequan.zhou@hatch.com

Hatch Ltd., 80 Hebron Way, Suite 100, St. John's, NL, Canada[2]
E-mail: diana.sankar@hatch.com

ABSTRACT

In dam safety assessments the Inflow Design Flood (IDF) may be the Probable Maximum Flood (PMF) according to the current regulations and guidelines in both the Canadian Dam Association (CDA) and the Lakes and River Improvement Act (LRIA) of Ontario. This IDF is set to be the PMF when there are extreme hazards imposed to downstream communities in the event of a dam failure. A dam must be designed (or be modified if an existing dam does not have the spillway capability) to safely pass the IDF. The peak flow, flood volume, the shape of the hydrograph and flood duration are important components of the PMF hydrograph. A PMF hydrograph, not just a PMF peak flow, is important when determining the hydrological risk of dams where a dam has significant storage capacity. Traditionally, these parameters of the PMF are obtained by rainfall-runoff modelling based on Probable Maximum Precipitation (PMP). The processes of the analyses may take considerable time and resources. This paper proposes an alternative approach using regional relationships based on streamflow statistics and envelope curves of peak flow and flood volume for Ontario, Canada. The approach is similar to the Hershfield (1961, 1965) method for the derivation of PMP. A database of 396 flow stations was analyzed to derive the relationship between the PMF peak flows and the corresponding volumes. A PMF hydrograph can be developed for ungauged locations within Ontario using the proposed method.

KEYWORDS: PMF, Flood, Hydrology, Dam Safety, Regional Technique, Flood Frequency, Risk, Hydrograph

1. Dequan Zhou, P. Eng. Hatch Ltd.
2. Diana Sankar, P. Eng. Hatch Ltd.

1 INTRODUCTION

As part of a dam safety assessment, an Inflow Design Flood (IDF) is used to determine the size of a spillway and the hydrological hazards of a dam. If a dam break event would lead to incrementally large losses of life, high social, cultural and economical damages and severe environmental consequences, the dam has a high to very high Hazard Potential Classification (HPC) (Ontario Ministry of Natura Resources, 2011). In this case, the IDF could be the Probable Maximum Flood (PMF) based on standard dam safety guidelines (CDA, 2013 and Ontario Ministry of Natural Resources, 2011). All dams must be able to safely pass the IDF to meet the safety requirement in a standard based regulation system.

Various methods exist for the derivation of IDF less than the PMF event. These approaches include: 1) rainfall-runoff modeling (FEH, 1999). 2) flood frequency analyses based on historical flood records (for dam's IDF much less than the PMF) (Moin, S. and M. Shaw (1985)), and 3) regional flood estimation techniques (Nathan, et. al., 1994, Abrahamson et. al., 2010, and Zhou, 2023).

The most popular method is through a Probable Maximum Precipitation (PMP) – Probable Maximum Flood (PMF) approach. The first step is to determine the PMP by site specific study. The PMP can be derived by regional PMP maps or by statistical (e.g. Hershfield, 1961, 1965) methods (WMO, 2009). The PMP will then be used in a rainfall-runoff model to obtain the PMF. Various hydrological models are used in rainfall-runoff simulations.

The rainfall-runoff modeling approach is considered a reliable after the model was calibrated and validated properly. However, this approach takes considerable time to undertake. The advantage of the rainfall-runoff simulation approach is that many of the processes are physically based and hence are supported by physical laws. This is also a disadvantage at the same time since lack of data may often lead to misuse of these models.

Flood frequency analysis is another option for IDF derivation. There are two obvious drawbacks in using the flood frequency analysis in deriving the IDF. The IDF has a recurrence interval at least in a period of hundreds, thousands or even millions (e.g. PMF) of years but the length of available historical data rarely extends past 100 years. Therefore, the frequency curve must be extrapolated to a length significantly longer than the data recording period. This will introduce significant errors and lead to unreliability in predictions of the magnitude of the IDF. The second drawback is that flood frequency analyses are often done only for the peak flows. However, the flood volumes, time to peak and durations are also important controlling factors that are needed in dam safety assessments (especially for dams with storage reservoirs). Univariate frequency analysis does not solve the full IDF problem. Due to the need for the other variables, multivariate frequency analysis techniques have been developed. The common approach is bivariate (peak flow and total flood volume) frequency analysis applying copulas techniques (Chowdhary et.al. 2011, Gaál et.al, 2015 and Zhou, 2006, 2015). This type of frequency analysis requires new definitions of return period in bivariate or multivariate conditions (Gräler et al 2013).

In recent years, some regional and empirical equations were developed to simplify the processes in PMF estimation. Nathan et. al. (1994) developed an equation for south-eastern Australia for the estimation of peak PMF flow. Abrahamson and Pentland (2009, 2010) developed a series of empirical equations for British Columbia streams. Zhou (2023) used 92 sites specific PMF in Ontario and derived an empirical equation for Ontario streams. Zhou and Sankar (2025) developed a PMF peak flow derivation procedure based on historical statistics similar to the Hershfield method for PMP. The regional approach provides an effective way for the determination of PMF peak flows.

These methods have the same disadvantage as univariate flood frequency analysis, only the peak flows are estimated. For small reservoirs or run-of-river dam sites, knowing the peak flow will be sufficient to determine the IDF peak water levels. When the storage capacity of a dam or reservoir is large enough to lead to certain levels of peak flow reduction, the peak flow, flood volume and the shape of the flood hydrograph will be important variables in the flood routing processes. Therefore, there is a need to estimate the corresponding flood runoff volume as well as the shape of the hydrograph in a full dam safety assessment.

This paper describes a procedure to derive the PMF hydrograph using historical recorded relationship between peak flows and the total flood volumes of extreme floods from 396 flow stations located in Ontario. The assessment of the dataset demonstrated that there are very strong correlations between peak flows and total runoff volumes in Ontario streams. The high dependence between the two variables established a solid foundation on the framework for derivation of IDF hydrographs.

2 DATABASE

In 2014, the Ontario Ministry of Natural Resources published a database of peak flows for 297 streamflow stations of the Great Lakes-St. Lawrence watersheds. The stations covered a large portion of Central and Southern Ontario, but excluded Northern Ontario. To make the dataset representative of the entire province an additional 99 stations in Northern Ontario were added into the database from the Hydrometric Historical Data (HYDAT) from Environment Canada, increasing the number of stations to 396. The database contained the variables of drainage area, maximum flood records, mean, standard deviation, coefficient of skewness and kurtosis for each station. All of the stations have at least 10 years of extreme flow data. In the database, the size of the drainage areas ranged from 0.07 km^2 to 118,000 km^2. The peak floods ranged between 0.36 m^3/s to 8270.0 m^3/s (Table 1).

The spatial distributions of the stations are plotted in Figure 1.

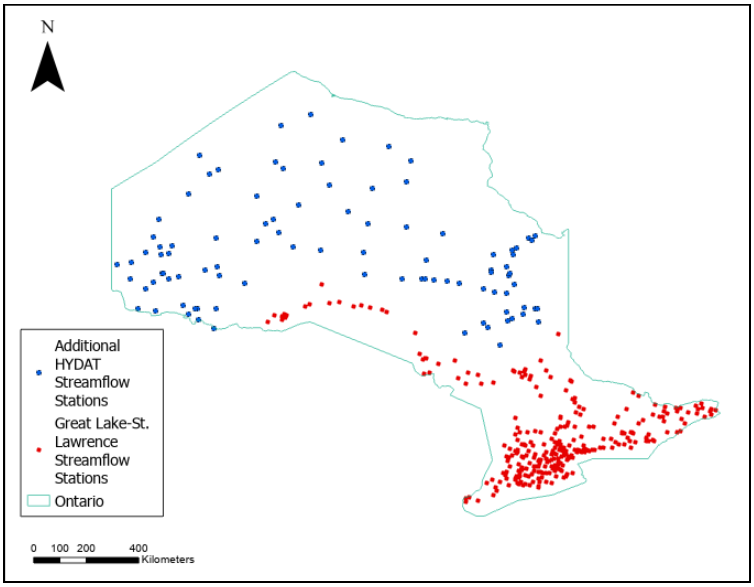

Figure 1 The spatial distribution of stations within the database

In Figure 1, the stations represented by red dots are the 297 Great Lake watersheds (Ontario, 2014). The 99 stations represented by blue triangles are the stations added based on HYDAT. As can be seen from Figure 1, the stations cover the province of Ontario well. The database contains historical maximum peak flows, but did not contain maximum flood volumes. For the purpose of this assessment, the maximum flood volumes were derived based on the following procedure:

1) Using the long-term streamflow records to find the year and date of the highest peak flow in the records of each station; 2) find the starting and the ending dates corresponding to the historical maximum flood; 3) identify the baseflow portion (as shown in Figure 2); 4) estimate the total flood runoff volume (with baseflow excluded). Figure 2 illustrates the baseflow separation from the flood volume estimate and the definitions of time to peak (T_p), flood duration (D) and the peak flow (Q_p) are also shown graphically.

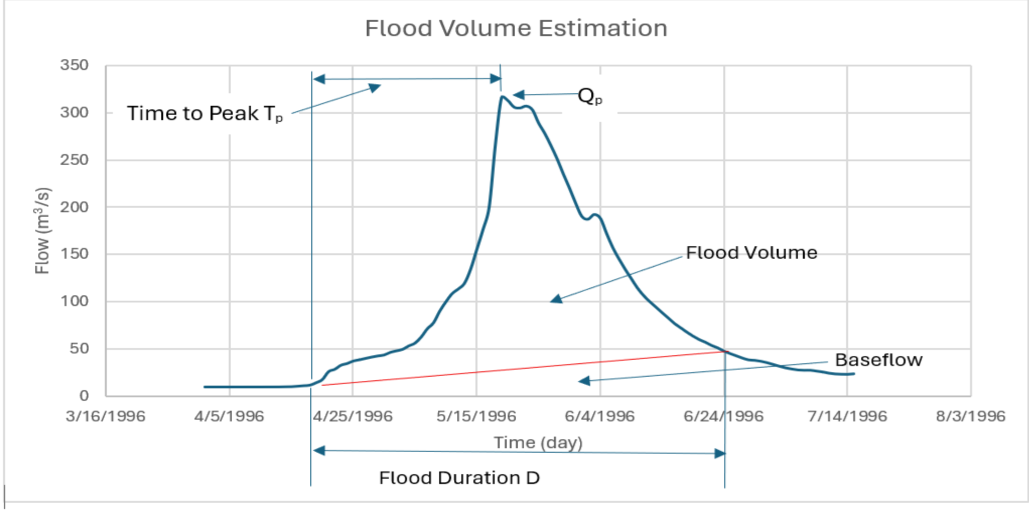

Figure 2 Determination of Flood Volume

The main statistics of the database are presented in Table 1. This database is used in the analysis of the relationship between the peak flow and total flood volume. Statistical analysis shows that the two variables are highly correlated with a R^2 of 0.79.

Table 1 Summary of Watershed Database

	Drainage Area	**Peak Flow**	**Flood Volume**	**T_p/D**
	km^2	m^3/s	Million m^3	
Min	0.1	0.36	.04	0.21
Max	118000.0	8270.0	17529.3	0.83
Mean	4043.4	410.7	575.8	0.41
StDev	12171.7	936.2	1973.7	0.11

Zhou (2024) used the database to derive a new method for predicting the PMF peak flows by the concept developed by Hershfield for PMP (1960, 1965). However, as discussed in the previous sections, a hydrograph is necessary to adequately assess the IDF when the dam impounds a reservoir with significant storage, and only a peak flow is not sufficient. The flood volume and flood shape are also important to describe the full hydrograph. The proposed approach to estimate the PMF hydrograph is described in Section 3.

3 METHODOLOGY

3.1 Peak Flow Estimation

The estimation of PMF peak flows can be obtained by three simplified methods in Ontario, such as 1) empirical equation (Zhou, 2023), 2) Statistical method (Zhou and Sankar 2025), and 3) Creager equation (Creager, 1945).

3.1.1 Empirical Equation

In 2023, Zhou developed an empirical equation for the estimation of PMF peak flows in Ontario using a regional approach. Based on 92 PMF peak flows estimated for dam sites in Ontario, the equation with the following form was obtained:

$$Q_{PMF} = 12.8312DA^{0.63564} \qquad (1)$$

Where Q_{PMF} is the PMF peak flow in m^3/s, DA is the drainage area in km^2. The regression equation has a $R^2 = 0.863$ and adjusted $R^2 = 0.861$. This equation has been accepted by the Ontario Ministry of Natural Resources (MNR) for the quick estimation of PMF peak flows for some of their dam sites. This empirical equation can be applied to sites where a quick peak of PMF is required and there are no available streamflow records.

3.1.2 Statistical Method

This method was developed for a site where there are long term flow records. Statistics of historical stream floods can be used for the estimation of PMF. The first step is to do the statistical assessment of the historical flood time series, the second step is to obtain the mean ($Q_{p,mean}$) and standard deviation (Std_p) of the peak flows, the third step is to estimate the frequency factor of the PMF (K_{PMF}). Zhou and Sankar (2025) found that this factor can be defined by an empirical equation calculated as a function of the historical average peak flow ($Q_{p,mean}$) of a site as follows:

$$K_{PMF} = 112.4514Q_{p,Mean}^{-0.34974} \qquad (2)$$

Where, K_{PMF} is the frequency factor for PMF estimation and $Q_{p,mean}$ is the mean peak flow (in m^3/s) estimated by the statistics of the site. Then the peak flow of the PMF can be calculated by:

$$Q_{PMF} = Q_{p,mean} + K_{PMF}Std_p \qquad (3)$$

Where Q_{PMF} is the peak flow of the PMF event (m^3/s), Std_p is the standard deviation of the peak flow statistics, and K_{PMF} is the frequency factor defined by equation (2).

3.1.3 Creager Equation

The Creager equation was derived by Creager et. al. (1945) and has been used as a method to validate the PMF peak flow used in many engineering works. Zhou (2023) found that all of the 92 PMF peak flows previously estimated in Ontario sites fall into the Creager curves with the coefficient C value between 5 to 45. For this reason, the peak flow estimated by the Creager equation with C value between 5 and 45 could be viewed as a good approximation of PMF peak flow for a site. The Creager equation has the following form:

$$Q_{PMF} = 46\,C\,DA^{0.984DA^{-0.048}} \quad \text{(original equation in imperial units)} \qquad (4)$$

Where drainage area DA is in $mile^2$ and Q_{PMF} is the peak of the PMF flow in ft^3/s.
This equation can be expressed as

$$Q_{PMF} = 1.303 * C * (0.386 * DA)^{0.936DA^{-0.048}} \quad \text{(in SI units)} \qquad (5)$$

Where DA is in km^2 and Q_{PMF} is in m^3/s.
Using an average C value of 25 can provide a reasonable first approximation of the Q_{PMF} in Ontario.

3.2 Relationship between Peak Flow and Flood Volume

As has been discussed previously, peak flow is only one of the variables that describes a PMF event. For dam safety assessment sites with storage reservoirs, the peak flow, flood runoff volume and the shape of the hydrograph are needed. Based on the database of 396 streamflow stations, the historical maximum flood peak flows and their corresponding flood volumes were obtained. A relationship between the maximum peak flows and the peak flood volumes was then derived as follows:

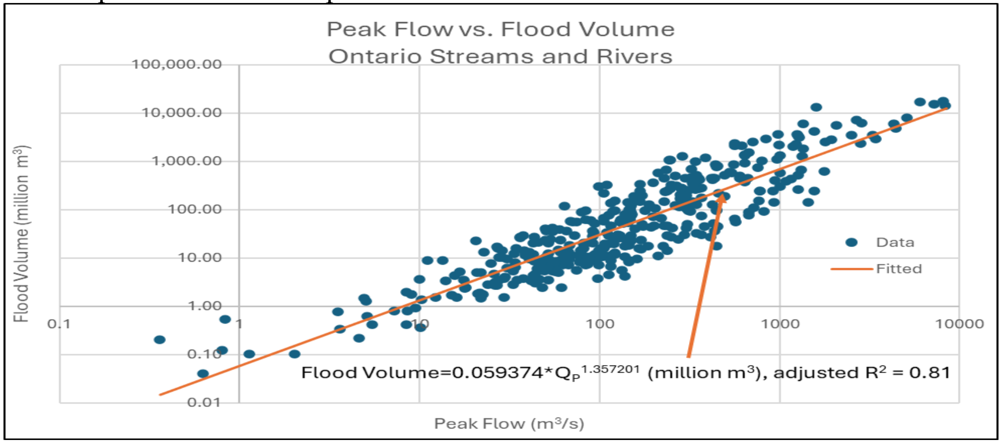

Figure 3 Relationships between historical maximum Peak Flow and Flood Volume (396 stations)

$$Vol_{p,Max} = 0.059374Q_{p,Max}^{1.357201} \qquad (6)$$

Where $Vol_{p,Max}$ is the peak flood volume in million m^3 and $Q_{p,max}$ in m^3/s for the historical maximum peak flood. The regression has a $R^2 = 0.9$ and an adjusted $R^2 = 0.81$ which means that there is a strong relationship between the peak flow and flood volume in Ontario streams. The relationship is plotted in Figure 3.

3.3 Time to Peak

Another important factor in the determination of PMF and IDF hydrographs is time to peak. This variable in principle is a function of drainage area, basin land use, density of channel network, the main channel geometry, slope, roughness and bed materials. A statistical analysis of time to peak to flood duration ratio, T_p/D, was performed for the 396 Ontario stream's historical flood events. The results are presented in Table 2 and Figure 4. Referring to Table 2, most of Ontario stream have T_p/D ratio between 0.4 to 0.45 (81 stations). More than 60% of the stations have T_p/D ratio between 0.25 to 0.50. This indicates that the peak floods are mainly front loaded (i.e. the peak flow occurs in the first half of the flooding process). This table can be used to guide the selection of T_p/D ratio in extreme flood analysis. Therefore, it is reasonable to select a T_p/D ratio between 0.25 to 0.50 for ungauged locations.

Table 2 Tp/D Ratio Distribution (396 Stations)

Tp/D range		Number of Stations	Percentage %	Cumulation %
0.20	0.25	27	6.8	6.8
0.25	0.3	56	14.1	21.0
0.30	0.35	46	11.6	32.6
0.35	0.4	63	15.9	48.5
0.40	0.45	81	20.5	68.9
0.45	0.5	62	15.7	84.6
0.50	0.55	17	4.3	88.9
0.55	0.6	23	5.8	94.7
0.60	0.65	9	2.3	97.0
0.65	0.7	9	2.3	99.2
0.70	0.75	2	0.5	99.7
0.75	0.8	1	0.3	100.0

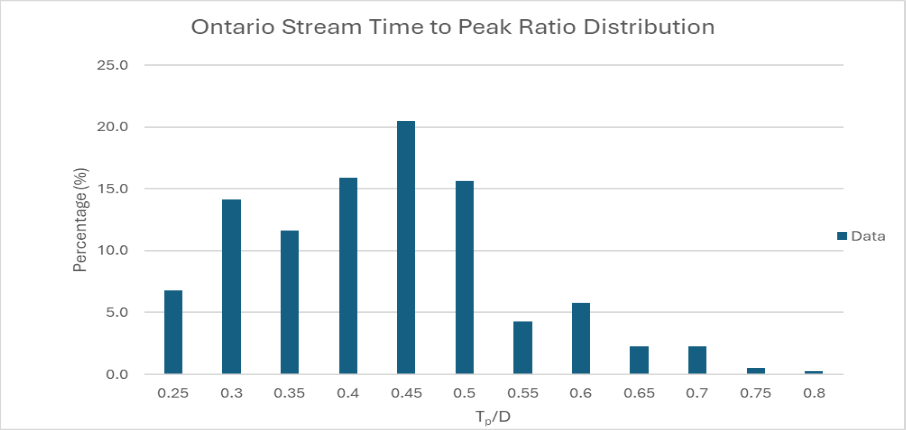

Figure 4 Time-to-Peak Distribution (396 Ontario Streams)

However, for gauged sites, the time-to-peak/flood duration (T_p/D) can be determined by historical data.

The total duration, D, can be estimated by dividing the flow volume by the peak flow using a triangular hydrograph, Figure 5.

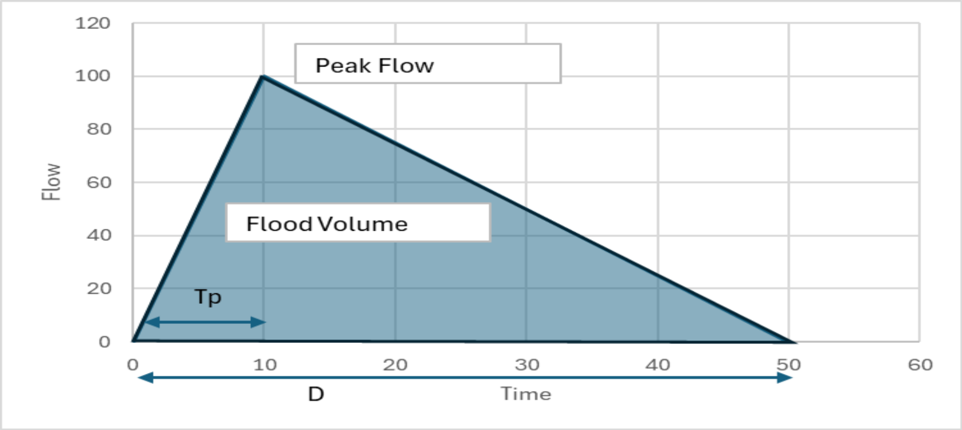

Figure 5: Derivation of D, Time of Flood Duration

3.4 PMF Hydrograph Derivation

In sections 3.1, 3.2, and 3.3 three of the important variables, peak flow Q_{PMF}, flood volume Vol_{PMF}, and T_p, time to peak were estimated. This section discusses the remaining steps for fully defining the shape of the PMF hydrograph using these variables.

There have been numerous studies in the development of design flood hydrographs. The use of statistical distribution functions in a synthetic design flood hydrograph has been widely accepted as a general way of representing design flood processes. Probability distributions employed for this purpose include Gamma distribution (Yue, et. al. (2002), Bhunya et. al. 2003), Weibull Distribution (Bhynya, et. al. (2007), Log-normal (Bhattacharjya, 2004), Beta distribution (Johnson and Kotz 1970), and other types of distributions. Rai and Singh (2008) summarized the method of parameter estimation for these probability distributions given the time-to-peak (T_p) and the peak flow (Q_p). All of these probability distributions can be applied for synthetic design flood hydrograph representations. However, based on the simulation results, the goodness-of-fit to the natural flood hydrographs to match the peak flow, total runoff volume and the shape by these probability distributions are not consistent. For some river's data, the fittings are acceptable. But for other rivers, the fittings are poor. This might be due to the fact that the parameter estimations are only based on time to peak (T_p) and peak flow (Q_p), while the total flood volume was not considered.

Reitz and Kreps (1945) developed a general hydrograph model that can be used for the estimation of design flood hydrographs. Their design hydrograph consists of two portions. For time steps before the time-to-peak (T_p), the hydrograph follows:

$$Q_t = Q_p sin^2(\frac{\pi t}{2T_p}) \qquad \text{for } t \leq T_p \qquad (7)$$

And for time after the time to peak T_p:

$$Q_t = \exp(-\alpha(t - T_p)) \qquad \text{for } t > T_p \qquad (8)$$

The Reitz and Kreps model has only one parameter α and α can be first estimated by the time to peak, peak flow and flood volume (Strupczewski et. al. 2011) as:

$$\alpha = \frac{2Q_p}{2Vol - Q_p T_p} \qquad (9)$$

Where Q_p is the peak flow (m^3/s), Vol is the flood volume (m^3) and T_p is in seconds or hour. Normally parameter α obtained by equation 9 requires adjustments given Q_p, T_p and Vol of the flood so that the integrated total volume under the hydrograph equal to the given volume (Vol). It was found that the Reitz and Kreps model can fit the general shape of historical maximum flood hydrographs in Ontario well. Figure 5 shows one example of the recorded and fitted flood hydrographs for station 02MB006. This station has flow records from 1970 to 2022. The historical maximum flood occurred in 2017 with a peak flow of 55.2 m^3/s (55.0 m^3/s after baseflow removal). The total flood volume in the 7 days (approximately 150 hours) is estimated to be about 1.04 million m^3. The time-to-peak occurred on the third day or about 50 hours after the beginning of the flood. Figure 6 presents the comparison of the recorded and fitted flood hydrographs.

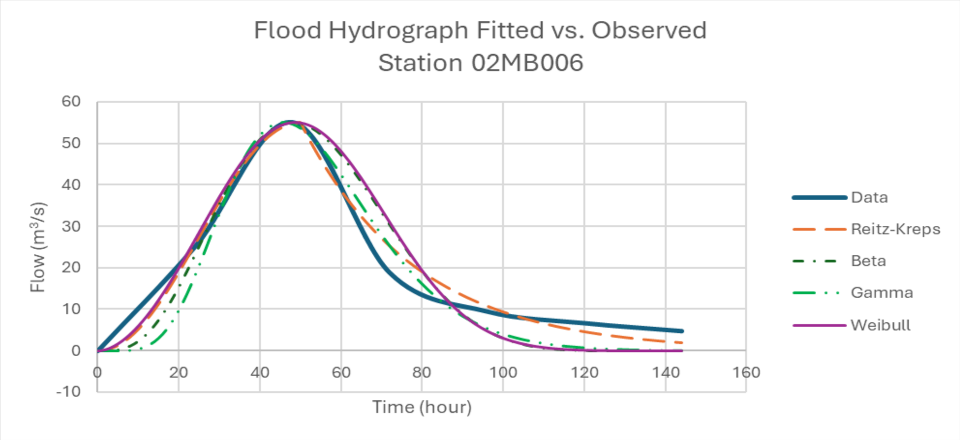

Figure 6 Comparison of Recorded and Fitted Historical Maximum Flood Hydrographs using Various Models

It can be seen that the shape of the flood hydrograph was simulated well by the Reitz and Kreps design hydrograph model, especially from the start of the hydrograph up to the peak flow. Between 62 hours to 108 hours the two hydrographs show some deviations, however the total volumes matched very well.

The flood hydrographs derived from equations 7, 8 and 9 require baseflow adjustments. Baseflow is not directly generated from the excess rainfall during a storm event. The baseflow portion can be estimated based on baseflow index (BFI). A constant baseflow during a flood event, see Figure 2 for an example, can be used and added into the PMF hydrograph.

The comparisons of goodness-of-fitting shown in Figure 6 show that the Reitz-Kreps model fits the data best. The other distributions do not work well for the tail area. But all of the models show reasonable agreement with the peak flow. Overall, the Reitz-Kreps model is recommended for use for streams in Ontario. However, the other types of models (Beta and Gamma) can be applied to create a flood hydrograph depending on conditions and response observed at the site of interest.

4 SUMMARY OF PMF DERIVATION

Based on the methodology described in Section 3, the PMF hydrograph derivation procedure is summarized below:

A) Determination of project drainage area,

B) Estimation of PMF peak flow. Depending on the availability of data, this can be undertaken by:

  B.1) If the site does not have any streamflow records, the PMF peak flow are calculated by Zhou's empirical equation (equation 1) or Creager Curve (equation 5)

  B.2 If the site has reliable streamflow records, the method described by Zhou and Sankar (2025) or equation 2 and equation 3, or the empirical equation and Creager equation

C) Estimate the flood runoff volume using equation 6

D) Estimate the time to peak T_p/D

E) Calculation of the PMF hydrograph using equations 7, 8 and 9. It shall be noted that equation 9 is only an approximation of the parameter α. α shall be adjusted by setting the difference between the given flood volume and the volume under the hydrograph to zero through changing α value by iterations.

The flood hydrograph developed following the procedure provides the full information for PMF flood (i.e. peak flow, time to peak, flood hydrograph for the entire duration.

5 CONCLUSIONS

In Ontario dam safety assessments, a PMF is needed determine the risk level of a dam to meet regulatory requirements. However, PMF determination has proven to be a difficult task with respect to technical effort and budget limitations. Currently, a lack of suitable methods to derive the PMF hydrograph is still a problem faced by the engineers.

In the past, there were a few methods for PMF peak flow estimation. However, peak flow alone does not fully define the PMF, especially for dam sites with considerable storage capacity. Other variables such as time to peak, shape of the flood hydrograph and the total volume of flood are also important.

This paper described a complete method to provide relatively accurate but quick techniques for PMF hydrograph development for Ontario streams, based on a database with 396 stations across the province.

An equation between peak flow and flood volume was constructed based on the data sets of the historical maximum floods.

Once both the peak flow and the flood volume are determined for the PMF event, a flood hydrograph can be calculated based on the Reitz and Kreps (1945) model or other suitable methods.

It shall be noted that the hydrograph needs to be adjusted by adding the baseflow into the time series.

A sensitivity analysis will help to determine the impacts of changing some of the variables, such as peak flow, flood volume and time-to-peak, in the PMF hydrograph to the safety of dams.

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